**Lattice planes and Miller indices**

**Lattice planes**

3 lattice vectors \( \{n_i \vec{a}_i, \ i = 1,2,3\} \) - which do not fall on a straight line - define a lattice plane, which is characterized by integers \((hkl)\):

**Miller indices**: \( h = m \cdot \frac{1}{n_1}, \ k = m \cdot \frac{1}{n_2}, \ l = m \cdot \frac{1}{n_3}, \ h, k, l \in Z \)

The reciprocal lattice vector \( \vec{G} = h\vec{g}_1 + k\vec{g}_2 + l\vec{g}_3 \) is perpendicular to the lattice plane \((hkl)\) and the distance between adjacent planes is: \( d_{hkl} = \frac{2\pi}{|\vec{G}|} \)

**Example: indices of planes in cubic lattices**

![Diagram of lattice planes and Miller indices](image-url)
Diffraction from periodic structures

The Laue condition:
Fourier expansion of scattering intensity \( \rho(R) = \sum_G \rho_G e^{iG \cdot R} \) yields scattering intensity: \( I(\Delta k) \propto \left| \frac{A_{out}}{R^2} \right|^2 \left| \sum_G \rho_G \int e^{i(G - \Delta k) \cdot r} dr \right|^2 \)
where the integral has significant contributions only for \( \Delta k = G \): → Laue condition: \( \int e^{i(G - \Delta k) \cdot r} dr = \begin{cases} V & \text{for } \Delta k = G \\ \sim 0 & \text{otherwise} \end{cases} \)
with intensity: \( I(\Delta k = G) \propto \left| \frac{A_0}{R^2} V^2 \right| \rho_G \left| \rho_G \right|^2 \)

Equivalence with Bragg law:
The condition \( |\Delta k = G| \) is equivalent to: \( 2k_0 \sin \Theta = |G_{hkl}| = \frac{2\pi}{d_{hkl}} \)
which yields the Bragg law:
\( 2d_{hkl} \sin \Theta = \lambda \)
for diffraction at the crystal planes (hkl)