A charged defect in a homogenous electron gas gives rise to "screening". What is the spatial extent of the perturbed electron distribution?

$\delta U(r)$ with $\delta U(r) \cdot e << E_F$: perturbation of a slowly varying local potential; $\delta U$ shifts $D(E)$ and an electron density $\delta n$ is distributed in the vicinity.

$\delta n = D(E_F) \cdot |e| \delta U$

Poisson equation:

$$\nabla^2 (\delta U(r)) = \frac{e^2}{\varepsilon_0} D(E_F) \delta U(r)$$

Spherical coordinates yield the solution

$$\delta U(r) = \alpha \frac{e^{-r/r_{TF}}}{r}$$

$r_{TF}$ is the Thomas-Fermi screening length with

$$r_{TF} = \sqrt{\frac{\varepsilon_0}{e^2 D(E_F)}}$$

For the free electron gas, $D(E_F) = \frac{3}{2} \frac{n}{E_F}$ and

$$r_{TF} \approx \frac{1}{2} \left( \frac{a_0^3}{n} \right)^{1/6}.$$ $D(E_F)$

Higher electron density leads to a shorter $r_{TF}$. For Cu, e.g., $n = 8.5 \cdot 10^{22} \text{ cm}^{-3}$ and $r_{TF} = 0.55 \text{ Å}$. 

**Chemical potential**

- Region of normal electron concentration
- Region of enhanced electron concentration

**Poisson equation**

- $0$
- $r$

**Separation $r/r_{TF}$**

- $0$
- $2$
- $4$
- $6$

**Energy E**

- $0$
- $n'$
- $n$
- $E_F$

**Potential energy $V(r)$**

- $\cdots -\frac{1}{r}$
- $-e^{-r/r_{TF}}$

**Density of states $D$**

- $0$
- $\delta U$
Above a critical electron density $n_C$ the screening length $r_{TF}$ becomes so small that electrons become delocalized → **metallic behaviour**

\[ n > n_C \Rightarrow r_{TF} < R_0/2, \]

\[ n < n_C \Rightarrow r_{TF} > R_0/2; \quad R_0 \text{ nearest neighbour distance of cores} \]

**Insulator** with localized electrons at low carrier density:

\[ r_{TF} \gg a_0, \text{ with Bohr radius } a_0 = 0.53 \, \text{Å}. \]

\[ r_{TF}^2 \approx \frac{1}{4} \frac{a_0}{n^{1/3}} \gg a_0^2 \]

**Mott's estimate:**  \[ n^{-1/3} \gg 4a_0 \]

**Metal-insulator transition:**

- Electrical conductivity $\sigma$ changes by orders of magnitude from metallic to insulating.
- Observed with increasing doping of a semiconductor by donor or acceptor atoms.
- The transition occurs at a concentration, where the ground-state wave functions of neighboring impurity atoms overlap.